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Regge behaviour of structure function and gluon distribution at low-x in leading order

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Abstract. We present a method to find the gluon distribution from the F_2 proton structure function data at low-x assuming the Regge behaviour of the gluon distribution function at this limit. We use the leading order (LO) Altarelli–Parisi (AP) evolution equation in our analysis and compare our result with those of other authors. We also discuss the limitations of the Taylor expansion method in extracting the gluon distribution from the F_2 structure function used by those authors.

1 Introduction

The measurements of the F_2 (proton and deuteron) structure functions by deep inelastic scattering (DIS) processes in the low-x region, where x is the Bjorken variable have opened a new era in parton density measurements [1]. It is important for understanding the inner structure of hadrons and ultimately of matter. It is also important to know the gluon distribution inside a hadron at low-x because gluons are expected to be dominant in this region. On the otherhand, the gluon distribution cannot be measured directly from experiments. It is, therefore, important to measure the gluon distribution $G(x,Q^2)$ indirectly from the proton as well as the deuteron structure functions $F_2(x,Q^2)$. Here the representation for the gluon distribution $G(x) = xg(x)$ is used, where $g(x)$ is the gluon density.

A few papers have already been published [2–9] in this connection. Here we present an alternative method to extract $G(x,Q^2)$ from the scaling violations of $F_2(x,Q^2)$ with respect to $\ln Q^2$, i.e. $\partial F_2(x,Q^2)/\partial \ln Q^2$. Our method is mathematically more transparent and simpler than those of other authors.

2 Theory

It is shown in [2,8] that the gluon distribution $G(x)$ at low-x can be obtained by analysing the longitudinal structure function. Similarly it is also shown in [3–7] that this distribution can be calculated from the F_2 proton structure function and its scaling violation. Moreover, in [9] we see that it is also possible to calculate the gluon distribution from the F_2 deuteron structure function and its scaling violation. The basic idea relies on the fact that the scaling violation of the F_2 structure function arises at low-x from the gluon distribution alone and does not depend on the quark distribution. As a demonstration of this fact, the scaling violation of the sea quark distribution as a function of x has been illustrated in [3]. Here as in Figs. 1a,b the scaling violation of the sea quark distribution using the KMRS B_- and B_0 parametrizations [10] are demonstrated, respectively. At low-x, actually already at $x = 10^{-2}$, the quarks can be neglected in the AP evolution for the number of flavours of $n_f = 4$.

Neglecting the quark the AP evolution equation for four flavours [3,4] gives

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} = \frac{5\alpha_s}{9\pi}\int_0^{1-x} G(x/(1-z),Q^2)P_{qg}(z)\mathrm{d}z, \quad (1)$$

where the LO splitting function is

$$P_{qg}(z) = z^2 + (1-z^2), \quad (2)$$

and α_s is the strong coupling constant.

Now, let $1-z = y \Rightarrow \mathrm{d}z = -\mathrm{d}y$. Again $z = 0 \Rightarrow y = 1$ and $z = 1-x \Rightarrow y = x$. Therefore (1) gives

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} = \frac{5\alpha_s}{9\pi}\int_x^1 G(x/z,Q^2)(2z^2-2z+1)\mathrm{d}z. \quad (3)$$

Now, let us consider the Regge behaviour of the gluon distribution [11]

$$G(x,Q^2) = Cx^{-\lambda(Q^2)}, \quad (4)$$

where C is a constant and $\lambda(Q^2)$ is the intercept. The Regge behaviour of the structure function in the large-Q^2 region reflects itself in the small-x behaviour of the

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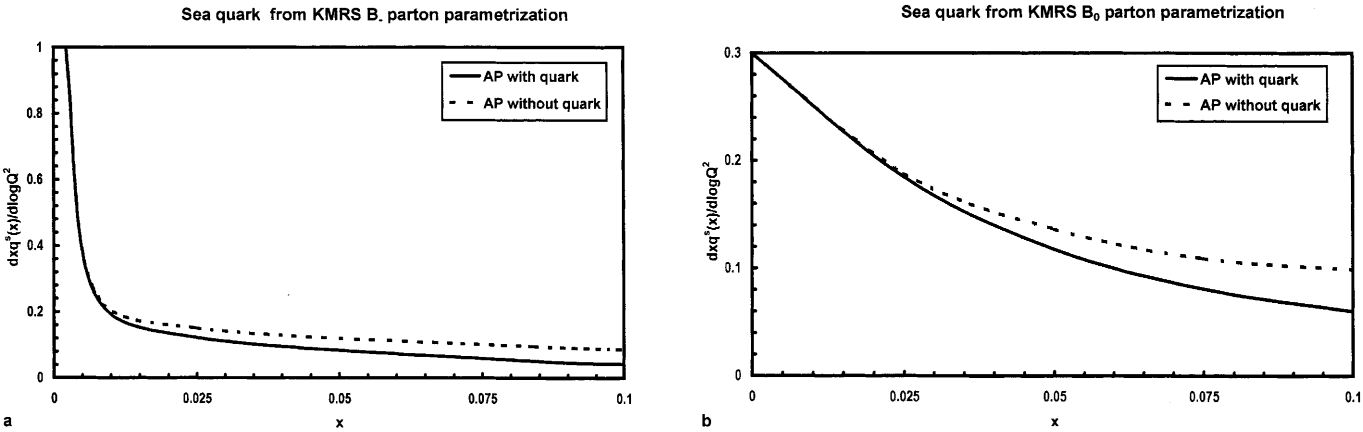

Fig. 1a,b. Scaling violations of sea quark distributions using the KMRS B_- and B_0 parametrizations [10] respectively as in [3]. The solid lines were obtained using the complete AP equations and the dashed lines were obtained neglecting quark distributions.

quark and the antiquark distributions. Thus the Regge behaviour of the sea quark and antiquark distribution for small-x is given by $q_{\mathrm{sea}}(x) \sim x^{-\alpha_P}$ corresponding to a pomeron exchange of intercept α_P=1. But the valence quark distribution for small-x given by $q_{\mathrm{val}}(x) \sim x^{-\alpha_R}$ corresponds to a reggeon exchange of intercept $\alpha_R = 1/2$. Since the same processes lead to gluon and sea quark distributions in the nucleon, we expect $G(x) \sim 1/x$. The x-dependence of the parton densities given above is often assumed at moderate-Q^2.

Applying (4) in (3) we get

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} = \frac{5\alpha_s}{9\pi} C \int_x^1 x^{-\lambda(Q^2)} z^{\lambda(Q^2)} (2z^2 - 2z + 1) \mathrm{d}z. \quad (5)$$

For fixed-Q^2, let $K(x) = \partial F_2(x,Q^2)/\partial \ln Q^2$ and $A = 5\alpha_s/(9\pi)$. Thus (5) gives

$$K(x) = ACx^{-\lambda(Q^2)} \int_x^1 (2z^{\lambda+2} - 2z^{\lambda+1} + z^{\lambda}) \mathrm{d}z. \quad (6)$$

Taking the logarithm and rearranging the terms (6) gives

$$\begin{aligned} \lambda = & \frac{1}{\ln x} \left[\ln \left\{ \frac{2}{\lambda+3}(1 - x^{\lambda+3}) \right. \right. \\ & \left. \left. - \frac{2}{\lambda+2}(1 - x^{\lambda+2}) + \frac{1}{\lambda+1}(1 - x^{\lambda+1}) \right\} \right] \\ & - \frac{1}{\ln x} \left[\ln\{K(x)/(AC)\} \right], \quad (7) \\ & \Rightarrow \lambda - \Phi(\lambda) = 0, \quad (8) \end{aligned}$$

where $\lambda \equiv \lambda(Q^2)$ and $\Phi(\lambda)$ represents the right hand side of (7). Now, (8) has been solved numerically using the iteration method [12] to calculate the values of $\lambda(Q^2)$ for different x-values for a fixed value of Q. A simple computer programme for this iteration method is given in Appendix A. Scaling violation of the F_2 structure function, i.e. $K(x) = \partial F_2(x,Q^2)/\partial \ln Q^2$, and the strong coupling constant at LO α_s are experimental inputs. C is the only free parameter in our calculation. After the calculation of $\lambda(Q^2)$ we can calculate $G(x,Q^2)$ from (4) for different values of the free parameter C and compare our results with those due to other authors.

Now, let us discuss the methods due to other authors. Prytz reported a method to obtain an approximate relation between the unintegrated gluon density and the scaling violations of the quark structure function at low-x at leading order (LO) [3] as well as at next-to-leading order (NLO) [4]. He expanded $G(x/(1-z))$ of (1) using the Taylor expansion formula at $z = 1/2$ to obtain the expression [3]

$$\begin{aligned} G\left(\frac{x}{1-z}\right) \approx & \, G\left(z = \frac{1}{2}\right) + \left(z - \frac{1}{2}\right) G'\left(z = \frac{1}{2}\right) \\ & + \left(z - \frac{1}{2}\right)^2 \frac{G"\left(z = \frac{1}{2}\right)}{2}, \quad (9) \end{aligned}$$

taking the derivative up to second order. This expression is then inserted in (1) and after integration one gets

$$\frac{\partial F_2(x)}{\partial \ln Q^2} \approx \frac{5\alpha_s}{9\pi} \frac{2}{3} G(2x) \quad (10)$$

for fixed-Q^2, which is the main result for the LO [3] analysis. Using a similar method he obtained the formula for the NLO [4] analysis,

$$\begin{aligned} \frac{\partial F_2(x)}{\partial \ln Q^2} \approx & \, G(2x) \frac{20}{9} \frac{\alpha_s}{4\pi} \left[\frac{2}{3} + \frac{\alpha_s}{4\pi} 3.58 \right] \\ & + \left(\frac{\alpha_s}{4\pi} \right)^2 \frac{20}{9} N(x,Q^2), \quad (11) \end{aligned}$$

where $N(x,Q^2)$ is given in [4].

Bora and Choudhury also presented a method [5] to find the gluon distribution from the F_2 proton structure function and its scaling violation at low-x using the Taylor expansion method. They also expanded $G(x/(1-z), Q^2)$ of (1) using the Taylor expansion method about $z = 0$ taking only the derivative up to first order in the expansion. While expanding they used only the first two terms in the infinite expansion series $x/(1-z) = x\sum_{k=0}^{\infty} z^k$ to get an expression. This expression is then inserted in (1) and after integration one gets

$$G(x_1, Q^2) \simeq \frac{9\pi}{5\alpha_s} \frac{A(x)+2B(x)}{[A(x)+B(x)]^2} \frac{\partial F_2(x,Q^2)}{\partial \ln Q^2}, \quad (12)$$

at

$$x_1 = x + \frac{B(x)}{A(x)+B(x)} x.$$

Sarma and Medhi also obtained a method [9] to find the gluon distribution from the F_2 proton and deuteron structure functions and their scaling violations at low-x. They also expanded $G(x/(1-z), Q^2)$ of (1) by using the Taylor expansion method taking only the derivative up to first order in the expansion. But unlike Bora and Choudhury method they considered the whole series $x/(1-z) = x\sum_{k=0}^{\infty} z^k$ to get the expression

$$G\left(\frac{x}{1-z}, Q^2\right) = G\left(x + x\sum_{k=1}^{\infty} z^k, Q^2\right) = G(x,Q^2) + x\sum_{k=1}^{\infty} z^k \frac{\partial G(x,Q^2)}{\partial x}. \quad (13)$$

Using this relation in (1) and then integrating one obtains for the proton

$$G(x_p, t) = \frac{9\pi}{5\alpha_s} \frac{1}{A(x)} \frac{\partial F_2^p(x,t)}{\partial t}, \quad (14)$$

and for the deuteron

$$G(x_d, t) = \frac{9}{5}\left[K_1(x) t \frac{\partial F_2^d(x,t)}{\partial t} + K_2 \frac{\partial F_2^d(x,t)}{\partial x} + K_3 F_2^d(x,t)\right], \quad (15)$$

where $x_p = x + B(x)/A(x)$, $x_d = x + D(x)/C(x)$ and $t = \ln(Q^2/\Lambda^2)$, Λ being the QCD cut-off parameter. Here $A(x), B(x), C(x), D(x), K_1(x), K_2(x)$ and $K_3(x)$ are some functions of x mentioned in [9].

Now, let us discuss the limitation of the Taylor expansion method in this regard. Applying the Taylor expansion [12] for the gluon distribution function in (1), we get

$$\begin{aligned} G\left(\frac{x}{1-z}, Q^2\right) &= G\left(x + x\sum_{k=1}^{\infty} z^k, Q^2\right) \\ &= G(x,Q^2) + x\sum_{k=1}^{\infty} z^k \frac{\partial G(x,Q^2)}{\partial x} \\ &+ \frac{1}{2}x^2\left(\sum_{k=1}^{\infty} z^k\right)^2 \frac{\partial^2 G(x,Q^2)}{\partial x^2} + O(x^3), \end{aligned} \quad (16)$$

where $O(x^3)$ are the higher order terms. Here we have $1-x < z < 0 \Rightarrow |z| < 1$ which implies that $x/(1-z) = x\sum_{k=0}^{\infty} z^k$ is convergent. In the previous methods, either the terms beyond second order [3,4] or beyond first order derivatives [5,9] of x are neglected in the expansion series (16). But in actual practice, this type of simplification is not possible because the contributions from the higher order terms cannot be neglected due to the singular behaviour of the gluon distribution.

There are some other methods also which are not based on the Taylor expansion method. Kotikov and Parente presented [7] a set of formulae to extract the gluon distribution function from the F_2 structure function and its scaling violation at small-x in the NLO approximation. They considered for singlet quark and gluon parton distributions $p(x,Q^2) \approx x^{-\delta_p(Q^2)}$ for a Regge-like behaviour and $p(x,Q^2) \approx \exp(0.5(\delta_p(Q^2)\ln(1/x))^{1/2})$ for double-logarithmical behaviour [6] where $p \equiv s, g$ and $\delta_s(Q^2) \neq \delta_g(Q^2)$. Then they put these distributions in the AP equations and solved for the gluon distribution by the standard moment method. Now for Regge-like behaviour, the gluon distribution becomes

$$g(x,Q^2) = \frac{1.14}{e\alpha(1+26.9\alpha)}\left[\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} + 2.12\alpha F_2(x,Q^2) + O(\alpha^2, x^{1-\delta}\right]. \quad (17)$$

for $\delta = 0.5$ and the number of flavours $f = 4$. Again for double-logarithmical behaviour the gluon distribution becomes

$$g(x,Q^2) = \frac{3}{4e\alpha}\frac{1}{(1+26\alpha[1/\tilde{\delta} - 41/13])} \times \left[\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} + O(\alpha^2 x)\right]. \quad (18)$$

A different method for the determination of the gluon distribution at small values of x has been proposed by Ellis, Kunszt and Levin [6] based on the solution of the AP evolution equations in the moment space up to next-to-next-to-leading order (NNLO). In this method the quark and gluon momentum densities are assumed to behave as x^{-w_0} where w_0 is a parameter the actual value of which must be extracted from the data. Here the gluon momentum density for four flavours is

$$xg(x,Q^2) = \frac{18/5}{P^{FG}(w_0)} \times \left[\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} - P^{FF}(w_0)F_2(x,Q^2)\right]. (19)$$

The evolution kernels P^{FF} and P^{FG} calculated in the $\overline{\text{MS}}$ scheme are expanded up to third order in α_s.

3 Results and discussion

We use HERA data taken by the H1 [13] and ZEUS [14] collaborations where the values of $\partial F_2(x,Q^2)/\partial \ln Q^2$ are listed for a range of x values at $Q^2 = 20\,\text{GeV}^2$. The recent HERA data are parametrized by the H1 [15] and

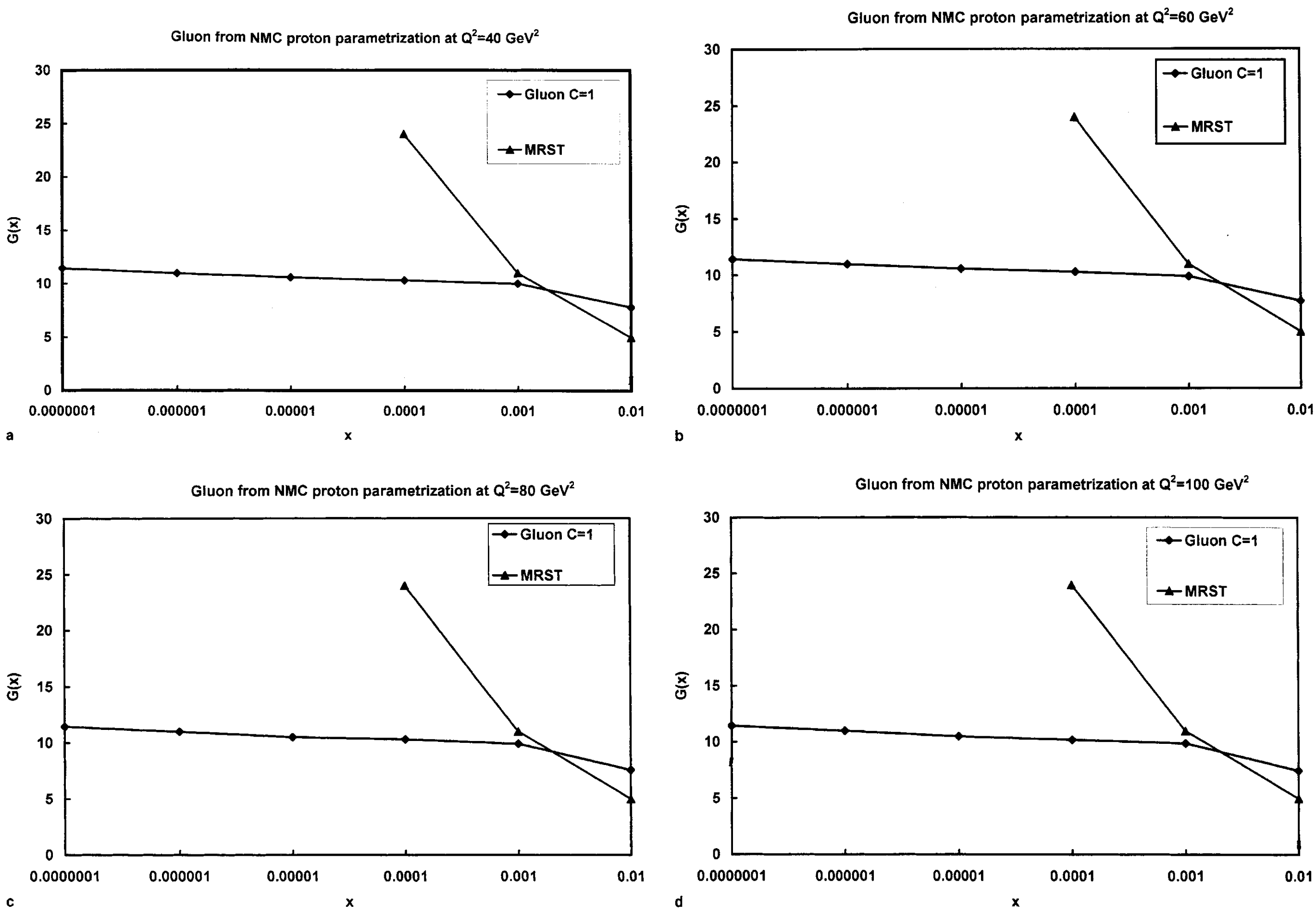

Fig. 2a,b. Gluon distribution $G(x)$ by our method from the NMC proton parametrization [17,18] at $Q^2 = 40, 60, 80$ and $100\,\mathrm{GeV}^2$ respectively with $C = 1$. In the same figure we include a global fit by MRST [21].

ZEUS [16] collaborations by some appropriate functions and we calculate $\partial F_2(x, Q^2)/\partial \ln Q^2$ at $Q^2 = 20\,\mathrm{GeV}^2$ for those functions also. We also use the parametrizations of the recent New Muon Collaboration (NMC) [17,18] F_2 proton structure function data from a 15-parameter function from which also we calculate $\partial F_2(x, Q^2)/\partial \ln Q^2$ at $40\,\mathrm{GeV}^2$. Now we apply the values of $\partial F_2(x, Q^2)/\partial \ln Q^2$ in (8) to calculate λ numerically by the iteration method [6] and hence the gluon distribution function $G(x, Q^2)$ for $C = 1$. We do not consider higher values of C, say $C = 100$, because in this case the neglect of the valence quark distribution $xq_{\mathrm{val}} \sim x^{1/2}$ is not so correct as the λ-value is close to -1/2 in quite a broad range of x. Moreover, in this case we obtain $xg \sim x^{1/2}$ and $xq_{\mathrm{val}} \sim x^{1/2}$. Then also we get $xq_{\mathrm{sea}} \sim x^{1/2}$. Otherwise it should not be neglected in (1). Then it is easy to obtain $F_2 \sim x^{1/2}$ which contradicts the experimental data. For our calculation the strong coupling constant α_{s} was taken from a NLO fit [19] to the F_2 data yielding $\alpha_{\mathrm{s}} = 0.180 \pm 0.008$ at $Q^2 = 50\,\mathrm{GeV}^2$ corresponding to $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 0.263 \pm 0.042\,\mathrm{GeV}$ and $\alpha_{\mathrm{s}}(M_{z^2}) = 0.113 \pm 0.005$. This value of α_{s} agrees with the one given by the Particle Data Group (PDG) [20]. But in our practical calculations we neglect the errors of α_{s} and Λ which are rather small.

We compare our result with the results of other authors discussed in the theory as well as the recent MRST global fit [21].

In Figs. 2a–d we present the gluon distributions $G(x)$ for different low-x values from the NMC proton data parametrization [17,18] at $Q^2 = 40, 60, 80$ and $100\,\mathrm{GeV}^2$ respectively. From the figures it is seen that the results are almost the same for all Q^2-values and $G(x)$ is slowly increasing when x decreases logarithmically. We also present the MRST global fit [21] result, but its rate of increment is much higher.

In Fig. 3 we present the gluon distributions $G(x)$ for different low-x values from the H1 HERA proton data [13] at $Q^2 = 20\,\mathrm{GeV}^2$. The middle line is the result without considering any error in the data. The upper and lower lines are the results with data adding and subtracting systematic and statistical errors with the middle values, respectively. As usual the gluon distribution $G(x)$ increases when x decreases. In the same graph we also present the $G(x)$ values for the MRST global fit [21] which is also increasing towards low-x values but with a somewhat smaller rate.

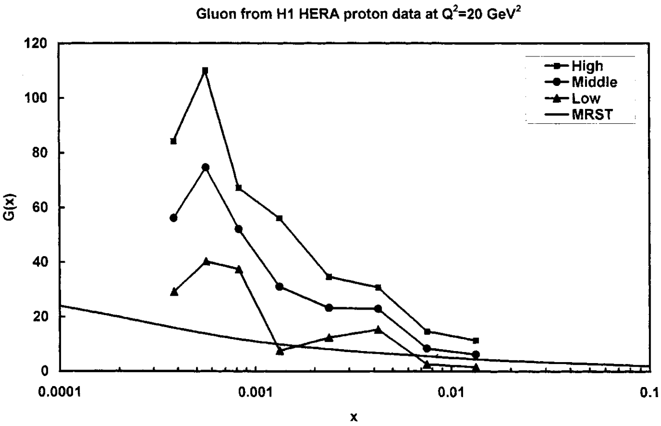

Fig. 3. Gluon distribution $G(x)$ by our method from the H1 HERA proton data [13] at $Q^2 = 20\,\mathrm{GeV}^2$ with $C = 1$. Here we present the results for the data (i) without considering the error (middle), (ii) adding algebrically statistical and systematic errors (high) and (iii) substracting algebrically statistical and systematic errors (low). In the same figures we include a global fit by MRST [21].

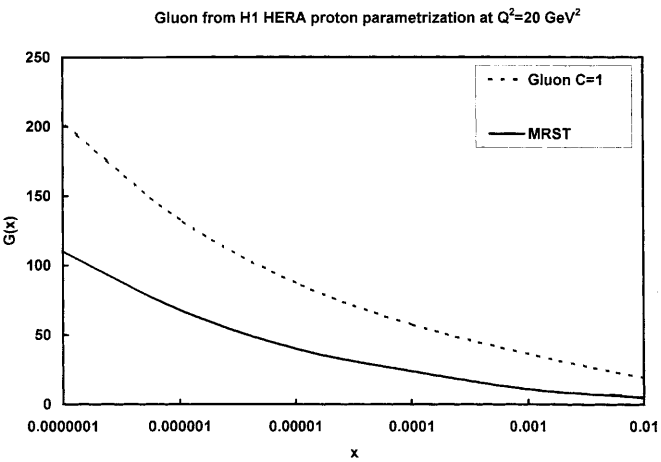

Fig. 4. Gluon distribution $G(x)$ by our method from the H1 HERA proton data parametrization [15] at $Q^2 = 20\,\mathrm{GeV}^2$ with $C = 1$. In the same figures we include a global fit by MRST [21].

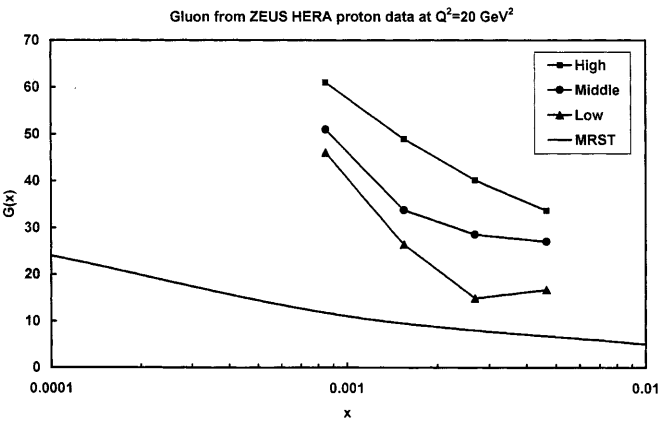

Fig. 5. Same result as in Fig. 3 from the ZEUS HERA proton data [14] at $Q^2 = 20\,\mathrm{GeV}^2$.

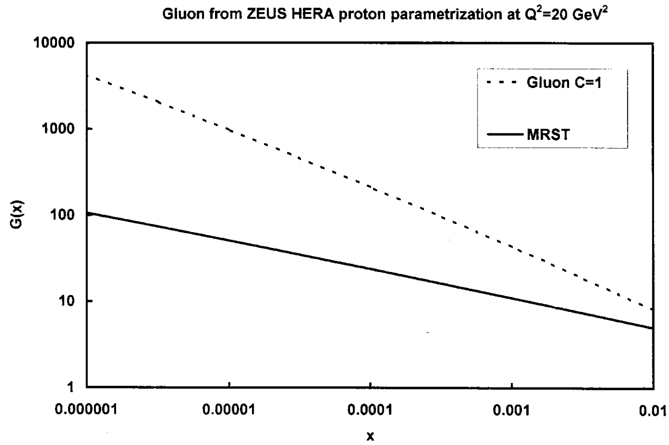

Fig. 6. Same result as in Fig. 4 from the ZEUS HERA proton data parametrization [16] at $Q^2 = 20\,\mathrm{GeV}^2$.

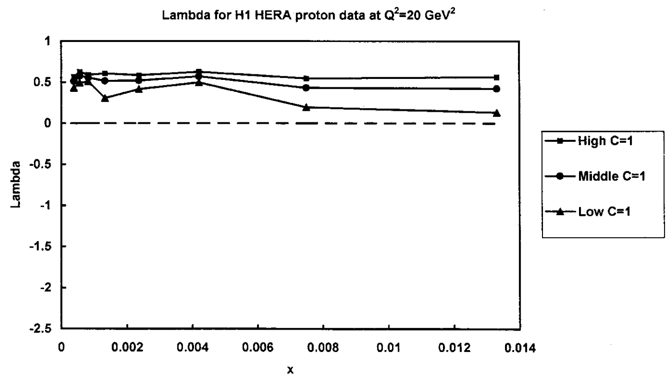

Fig. 7. λ-values by our method from the H1 HERA proton data [13] at $Q^2 = 20\,\mathrm{GeV}^2$ with $C = 1$. Here we present the results for the data (i) without considering the error (middle), (ii) adding algebrically statistical and systematic errors (high) and (iii) subtracting algebraically statistical and systematic errors (low).

In Fig. 4 we present the gluon distributions $G(x)$ for the H1 HERA proton parametrization [15] at $Q^2 = 20\,\mathrm{GeV}^2$ for different low-x values. The gluon distribution $G(x)$ is increasing when x is decreasing. In the same graph we present the $G(x)$ values for the MRST global fit [21], which is also increasing towards low-x values with a somewhat smaller rate.

In Fig. 5 we present the gluon distribution $G(x)$ ZEUS HERA proton data [14] at $Q^2 = 20\,\mathrm{GeV}^2$ for different low-x values. The descriptions and the results are the same as the H1 HERA data [13] depicted in Fig. 3.

In Fig. 6 we present the gluon distributions $G(x)$ for the ZEUS HERA proton parametrization [16] at $Q^2 = 20\,\mathrm{GeV}^2$ for different low-x values. The descriptions and the results are the same as the H1 HERA parametrization [15] depicted in Fig. 4.

In Fig. 7 we present the value of λ (Lambda) for the H1 HERA proton data [13] for low, middle and high values at $Q^2 = 20\,\mathrm{GeV}^2$ for different low-x values. All the graphs are almost parallel and the λ-values tend to ~ 0.5 at lower-x. That is, the parameter λ has a small dependence on x and

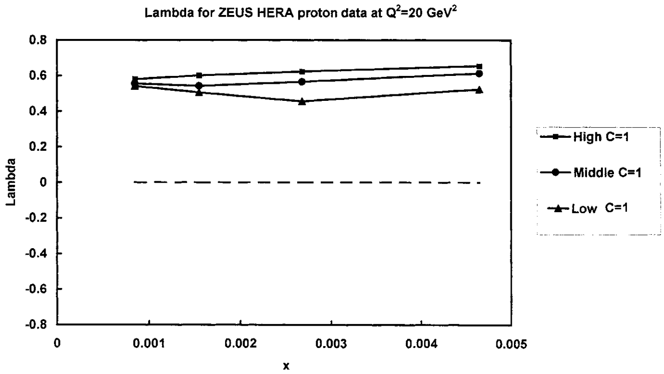

Fig. 8. Same result as in Fig. 7 from the ZEUS HERA proton data [14] at $Q^2 = 20\,\mathrm{GeV}^2$.

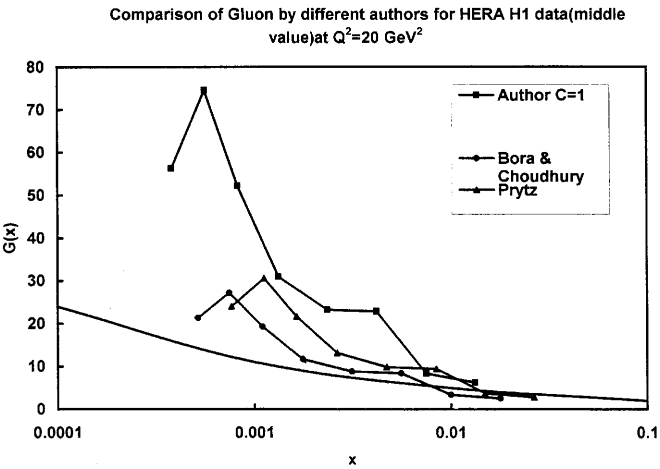

Fig. 9. Comparison of gluon distribution $G(x)$ from the H1 HERA proton data [13] by our method for $C = 1$ with those by other methods due to Bora and Choudhury [5] and Prytz [3]. In the same figure, we include a global fit by MRST [21].

Q^2. This behaviour is in good agreement with experimental data [22], fits [21,23] and with the double-logarithmical semi-analytical analysis [24–26].

In Fig. 8, we present the λ-values for the ZEUS HERA proton data [14] in the same way as in Fig. 7 and the analysis is also the same. For all the graphs λ values tend to ~ 0.5 as we approach a lower-x from some higher values of x.

In Fig. 9, we compare our results for the HERA H1 data (middle value only) [13] at $Q^2 = 20\,\mathrm{GeV}^2$ with those of Bora and Choudhury [5] and Prytz [3]. In the same figure, we also present the result for the MRST global fit [21]. For all cases the gluon distribution $G(x)$ is increasing when x is decreasing but with different rates. The rates of increment in our result is highest and in MRST is lowest.

4 Summary and conclusion

In this paper we present an alternative method [2–9] to extract the gluon distribution $G(x, Q^2)$ from the scaling violation of the F_2 proton structure function $\partial F_2(x)/\partial \ln Q^2$ at low-x. We compare our result with those of other methods due to Bora and Choudhury [5] and Prytz [3], and with a global fit due to MRST [21]. The gluon distribution will increase as usual when x decreases.

We discussed the limitations of the Taylor expansion method [12] in calculating the gluon distribution from the scaling violation of the F_2 structure function at low-x. Prytz in both LO [3] and NLO [4] and Bora and Choudhury in LO [5] used this method to extract the gluon distribution from the scaling violation of the F_2 structure function at low-x in a slightly different way. But all the authors neglected the higher order terms in the Taylor expansion series, which is not a good approximation for the singular behaviour of the gluon distribution at low-x, because the contributions from the higher order terms in the series are not negligible. Sarma and Medhi [9] used this method in some improved way with a better approximation; yet the basic approximation of neglecting higher order terms in the expansion series could not be avoided. On the other hand in the Kotikov and Parente method [7,8] also these authors approximated their results by neglecting some higher order terms. Moreover, their method is to some extent complicated. The Ellis, Kunszt and Levin method [6] neither has been more developed than other methods. Though their analysis is up to NNLO, the kernels are parameter dependent and the x-ranges are lower than the HERA region. In the present method of course we use a free parameter C; yet the other ambiguities due to the approximation of the Taylor expansion series can be avoided. Moreover, our method is very simple and the computer programme can calculate the gluon distribution immediately when we put in the value of the scaling violation from experiment.

We can use this method by assuming a double-logarithmical behaviour [7] of the gluon distribution at low-x also. The present procedure is a LO analysis only. But there is a possibility to extend this method to NLO or higher to have more accurate results.

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Appendix

A simple FORTRAN programme for the calculation of λ from the scaling violation of the structure function is given here:

```
C       GLUON DISTRIBUTION FROM SCALING VIOLATION OF PROTON DATA
05      REAL Y, K, C, X, A, PHIX1, PHIX2, PHIX3, PHIX, P, AB, G
10      PRINT*, "Y=?"
15      READ*, Y
20      PRINT*, "K=?"
25      READ*, K
30      PRINT*, "C=?"
35      READ*, C
40      X=.3
45      ALPH=.118
50      PI=3.1416
55      A=(5.*ALPH)/(9.*PI)
56      PHIX1=2./(X+3.)*(1.-Y**(X+3.))-2./(X+2.)*(1.-Y**(X+2.))
57      PHIX2=1./(X+1.)*(1-Y**(X1.))
58      PHIX3=ALOG(K/(A*C))
60      PHIX=1./ALOG(Y)*(ALOG(PHIX1+PHIX2)-PHIX3)
65      P=X-PHIX
70      AB=ABS(P)
75      G=C*(Y**(-PHIX))
80      IF (AB .LT. .00000001) THEN
               PRINT*, C, Y, PHIX, G
        GOTO 10
        ELSE
               X=PHIX
        ENDIF
        GOTO 56
85      END
```

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